EFFECT OF NUMBER STRUCTURE AND NATURE OF QUANTITIES ON SECONDARY SCHOOL STUDENTS' PROPORTIONAL REASONING

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Abstract: This study investigates the effect of the number structure (integer vs. non-integer ratios) and the nature of the quantities (discrete vs. continuous) on the performance of secondary school students and on the strategies they use when solving proportional and non-proportional word problems. 551 secondary school students from 1st to 4th grade solved a test with proportional and non-proportional problems in which the task variables mentioned before were manipulated. Results showed that number structure affected students' performance and the strategies were used differently for proportional and non-proportional problems along the grades. However, the nature of the quantities showed no influence.

Key words: additive strategy, nature of quantities, number structure, over-use of proportionality, proportional reasoning

Previous research has shown that proportional reasoning implies not only understanding the multiplicative relationship between quantities that represent a proportional situation, but also the ability to discriminate proportional and non-proportional situations (Christou, Philippou, 2002; Cramer, Post, Currier, 1993; De Bock et al., 2007). This last point is very important because people often seem to behave as if proportionality is always applicable. This phenomenon of the use of proportionality in non-proportional situations has been shown and studied over the past few years in various domains (Fernández, Llinares, Valls, 2008; Modestou, Gagatsis, 2007; Van Dooren et al., 2005; for a review see Van Dooren et al., 2008). Van Dooren et al. (2005) observed that the students' performance on proportional word problems considerably improved from 3rd to

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6th grade of primary school. But they also observed that during the same years, students' tendency to erroneously over-use proportional methods in non-proportional problems also increased accordingly: Whereas in 3rd grade 30% of all non-proportional problems were answered proportionally, this percentage increased up to 51% in 6th grade.

On the other hand, research on proportional reasoning points to many task characteristics that influence students' strategies and performance in proportional problems (e.g., Harel, Behr, 1989; Tourniaire, Pulos, 1985). The current study investigates the development of the effect of two of these task characteristics throughout secondary school: the number structure (i.e., whether ratios are integer or non-integer) and the context factor "nature of the quantities involved" (i.e., whether quantities are discrete or continuous).

The effect of the appearance of integer or non-integer ratios in proportional problems has been extensively investigated. For instance, Karplus, Pulos, and Stage (1983) gave 6th and 8th graders a series of "lemonade puzzles" about the taste of two mixtures of sugar and lemon juice. Their study considered comparison problems (where the rates are given and they have to be compared) and missing value problems (where three quantities are given and the fourth quantity is unknown). In some problems the ratios between the given numbers were integer, while in others they were non-integer. The results indicated that the occurrence of non-integer ratios contributed considerably to the difficulty level of a proportional problem: When one or both ratios (internal [relationship between quantities of the same nature] or external [relationships between quantities of different nature]) were non-integers, fewer correct answers and more incorrect additive answers were given. Pupils' age also had an impact: 8th graders performed better than 6th graders, particularly on the problems with non-integer ratios. But, for the problems with integer ratios, differences were smaller or even non-existent. Van Dooren, De Bock, Evers, and Verschaffel (2009) investigated whether the numbers appearing in word problems are part of the superficial cues that lead Flemish primary school students (4th to 6th) to (over)use proportionality using proportional and non-proportional problems. An example of a non-proportional problem is: "Ellen and Kim are running around a track. They run equally fast but Ellen started later. When Ellen has run 16 laps, Kim has run 32 laps. When Ellen has run 48 laps, how many has Kim run?" For proportional problems, Flemish students' performances decreased as a result of non-integer ratios, and for non-proportional problems, non-integer ratios led to a decrease in the over-use of proportionality. These effects of the number structure diminished with increasing age: In the older primary school students, there was an overall tendency to apply proportional methods, regardless of the presence of integer or non-integer ratios.

Another task characteristic affecting students' proportional reasoning is the nature of quantities involved in the problem. In their review of the literature, Tourniaire and Pulos (1985) suggest that people can more easily visualize discrete quantities than continuous ones and therefore will perform better on proportional problems involving discrete quantities. However, several later studies have found contrary results. Jeong, Levine, and Huttenlocher (2007) found that 6-, 8- and 10-year olds used erroneous additive counting strategies more often in discrete situations and correct multiplicative strategies more in continuous situations. So students were more successful with continuous situations than with discrete situations. Also Boyer, Levine, and Huttenlocher (2008) found that 6-9 year old children had much more difficulty solving proportional reasoning problems when the proportions were represented with discrete units than problems for which the fractions were represented with continuous amounts. Using comparison problems, they found that children's difficulties were at least partly due to their propensity to compare quantities on the basis of the number of elements rather than on the basis of proportional relations.

Taking into account these two lines of research (the over-use of proportionality for non-proportional situations and the effect of tasks variables on students' performance in proportional problems), the present study investigates the effect of two task variables on secondary school students' performance when solving proportional and non-proportional problems and how these effects vary with age: 1) the presence of integer or non-integer ratios and 2) the continuous or discrete nature of quantities. Importantly, our study aims at extending the insight in the effect of these task variables on students' reasoning also to *non-proportional* situations and more particularly on students' tendency to over-use proportional strategies in such situations (De Bock et al., 2007; Van Dooren et al., 2009).

For the present study we focus on one specific type of non-proportional situations called "additive problems" by Van Dooren et al. (2005) because according to the literature, additive reasoning is a very common error in proportional reasoning (Hart, 1984; Misailidou, Williams, 2003; Tourniaire, Pulos, 1985). So, it is relevant to compare students' solution behavior on proportional problems (where proportional reasoning is correct and incorrect additive strategies are expected), on the one hand, and on the additive problems (where additive strategy is correct while erroneous proportional strategies are expected), on the other hand. We study how the two task characteristics affect students' solution strategies and performance on both kinds of problems. Furthermore, we focus on proportional and additive problems formulated in a missing-value format, because so far the effect of the variables has been studied mostly with tasks in a comparison format.

METHOD

Participants

Participants were 558 secondary school students: 124 students in the 1st grade of secondary school (12-13 year olds), 151 in the 2nd grade of secondary school (13-14 year olds), 154 in the 3rd grade of secondary school (14-15 year olds), and 129 in the 4th grade of secondary school (15-16 year olds) from 2 different Spanish schools. The number of boys and girls in each age group was approximately the same. One participating school was situated in a large and touristic city, the other in a small village. Pupils were from mixed socio-economic backgrounds.

According to the curriculum, in the 1st and 2nd grade of secondary school, students should be able to identify proportional relations between numbers from the analysis of tables, and use the Rule of Three algorithm (mechanistic algorithm – cross product) to calculate the missing values in proportional situations. In the 3rd and 4th grade, the focus is on studying proportionality from a functional perspective (linear functions).

Instrument and Procedure

We used a test with 12 word problems: 8 experimental problems (4 proportional [P] and 4 additive [A] ones) and 4 buffer problems that were included to prevent the students from discovering the experimental design. Examples of additive and proportional problems are given in Table 1. Following Van Dooren et al. (2005) we use the term additive problems to denote non-proportional situations modeled by $f(x) = x + b$ (with $b \neq 0$), and proportional problems to denote situations modeled by $f(x) = ax$, (with $a \neq 0$). All word problems were presented in a missing-value form.

Starting from 8 discrete situations (like loading boxes or manufacturing dolls) and 8 continuous situations (like skating or swimming certain distances), proportional or additive word problems were created by manipulating the crucial second sentence (for example, "They started together but Rachel skates faster" in the proportional problem, and "They skate equally fast but Rachel started earlier" in the additive one). We also manipulated the number structure in the problems. The ratios in proportional situations and the multiplicative relationships between numbers in non-proportional problems were ei-

Table 1. Examples of problems and manipulation of the nature of quantities (versions D and C) and number structure (versions I and N)

	Examples	I	N
P-D	Peter and Tom are loading boxes in a truck. They started together but Tom loads faster. When Peter has loaded a boxes, Tom has loaded b boxes. If Peter has loaded c boxes, how many boxes has Tom loaded?	40 160 80 Prop: 320 Addit: 200	40 100 60 Prop: 150 Addit: 120
A-D	Peter and Tom are loading boxes in a truck. They load equally fast but Peter started later. When Peter has loaded a boxes, Tom has loaded b boxes. If Peter has loaded c boxes, how many boxes has Tom loaded?	40 160 80 Prop: 320 Addit: 200	40 100 60 Prop: 150 Addit: 120
P-C	Ann and Rachel are skating. They started together but Rachel skates faster. When Ann has skated a m, Rachel has skated b m. If Ann has skated c m, how many meters has Rachel skated?	150 300 600 Prop: 1200 Addit: 750	80 120 200 Prop: 300 Addit: 240
A-C	Ann and Rachel are skating. They skate equally fast but Rachel started earlier. When Ann has skated a m, Rachel has skated b m. If Ann has skated c m, how many meters has Rachel skated?	150 300 600 Prop: 1200 Addit: 750	80 120 200 Prop: 300 Addit: 240

Numbers are schematically represented as a b
c x (Prop: Proportional answer, Addit: additive solution)

ther both integer or both non-integer. So, 2 of the 4 proportional problems and 2 of the 4 additive problems referred to discrete quantities (one with integer ratios between given numbers, D-I, and one with non-integer ratios, D-N). The other 4 referred to continuous quantities (again 2 with integer ratios, C-I, and 2 with non-integer ratios, C-N). With the obtained set of problems (64 experimental problems and 4 buffer problems), 8 parallel tests were composed, with 4 different orders (Table 2).

To exemplify the possible impact of the number structure, consider the proportional problems in Table 1. The P-D-I-version could be correctly solved by focusing on the external ratio (160 / 40)*: Tom has loaded 4 times as many boxes as Peter (160 = 4 × 40), so when Peter has loaded 80 boxes, Tom has loaded 80 × 4 = 320 boxes*. In the case of using the internal ratio *(80 / 40)*, a solution could be: *Peter has loaded twice as many boxes as initially (80 = 40 × 2), so Tom has loaded 160 × 2 = 320 boxes*.

An incorrect additive strategy could be: The difference between the number of boxes loaded by Tom and Peter is 160 – 40 = 120 then when Peter has loaded 80 boxes, Tom has loaded 80 + 120 = 200 boxes.

For the P-D-N-version, the correct strategy is a bit more difficult, since one has to deal with non-integer ratios. Using an external ratio approach would be: *Tom has loaded 100 / 40 (or 2.5) times the boxes of Peter, so when Peter has loaded 60, Tom has loaded 60 × (100 / 40) = 150 boxes.* For the internal ratio approach it would be: *the ratio of boxes loaded by Peter is 60 / 40, so the ratio of boxes loaded by Tom is 100 × (60 / 40) = 150 boxes.* An incorrect additive strategy in this problem could be: *As the boxes difference loaded by Peter and Tom is 100 – 40 = 60, then when Peter loads 60, Tom would load 60 + 60 = 120.*

In the same way, we can exemplify the impact of the discrete or continuous nature of the quantities in proportional problems using the problems P-D-N and P-C-N in Table 2. This impact could be expected particularly when students apply the so-called “unit approach” (a particular case of the external ratio approach) to a non-integer word problem. For example, a correct strategy for the P-C-N problem using such a unit approach would be: *If Ann has skated 1 meter, Rachel has skated (120 : 80 =) 1.5 meters. So if Ann has skated 200 meters, Rachel has skated (200 × 1.5=) 300 meters*. But for the

Table 2. Different task orders

Order	Number of P-A and A-P task switches
PDN-ACI-Buffer-ADI-PCI-Buffer-PDI-PCN-Buffer-ADN-ACN-Buffer	P-A: 2 A-P: 1
ACI-Buffer-ACN-PDN-Buffer-PDI-ADI-Buffer-ADN-PCI-Buffer-PCN	P-A: 1 A-P: 2
Buffer-PCI-PCN-Buffer-ADI-PDI-Buffer-ACI-ACN-Buffer-PDN-ADN	P-A: 3 A-P: 2
Buffer-ACN-ADI-Buffer-PDN-ADN-Buffer-PCI-PCN-Buffer-ACI-PDI	P-A: 2 A-P: 3

P-D-N-version, the correct strategy using the unit approach would be: *If Peter has loaded 1 box, Tom has loaded (100 : 40 =) 2.5 boxes. So if Peter has loaded 60 boxes, Tom has loaded (60 × 2.5 =) 150 boxes*. Of course, at the intermediate step, such a solution has no contextual meaning when dealing with discrete quantities because you cannot load half a box.

Furthermore, all experimental problems were controlled for number size (numbers with one or two digits), calculation complexity (e.g., the outcome is always an integer), the context (always actions), and the position of the unknown quantity (the unknown quantity is always the second magnitude that appears in the word problem).

Students had 50 minutes (i.e., the duration of a regular mathematics lesson) to complete the test. There were no further test instructions except allowing students to use calculators and asking them to write down the operations they had computed by means of the calculator.

Analysis

Answers to the experimental problems were classified as proportional (Prop, when the answer was achieved by relying on a proportional method), additive (Add, when an additive method was applied to find the answer) and other answer (Oth, when another wrong solution procedure was followed). For proportional problems, approaches where a proportional method was used are the correct approaches (we scored them with a 1), while answers obtained by additive or other methods are the incorrect approaches (we scored them with a 0). For additive problems, answers obtained by additive methods are correct (we scored them with a 1) while answers obtained by proportional or other methods are incorrect (we scored them with a 0).

In the rare cases wherein a student made a purely technical calculation error but the kind of calculations performed by the student could still unequivocally be categorized as proportional or additive (e.g., in the P-D-I problem [Table 1], the response 160 : 40 = 4 and then 4 × 80 = 360 was categorized as proportional), the answers were scored as such.

Answers were statistically analyzed by means of a repeated measures logistic regression analysis.

HYPOTHESES AND RESEARCH QUESTIONS

Based on the literature, we make the following predictions and pose research questions about students' responses to proportional and additive problems.

First, we expect that students will be more successful in proportional than additive word problems (HYPO 1). From previous research (Van Dooren et al., 2005), we know that Flemish primary school students (3rd to 6th grade) improved their performance on proportional problems but also increased the over-use of proportional methods in non-proportional problems; so we expect that this trend continues during secondary school.

With regard to number structure, we expect, in line with the proportional reasoning literature (Karplus et al., 1983), that secondary school students will be more successful in integer proportional problems than non-integer ones (HYPO 2A) and will use the additive strategy more frequently in non-integer proportional problems than integer ones (HYPO 2B). For additive problems, we expect that students will be more successful in non-integer problems than integer ones (HYPO 2C) and will use more proportional strategies in integer than non-integer problems (HYPO 2D). These predictions are made in line with Van Dooren et al. (2009), even though our participants are secondary school students. Also, we predict that the above-mentioned effects will gradually decrease from 1st to 4th grade of secondary school (HYPO 2E).

Given that the research on the variable nature of quantities shows contradictory empirical evidence, we did not raise specific hypotheses, but we will focus on the following questions: First, in proportional problems will students be more successful with continuous quantities or with discrete quantities (Q1)? Second, in additive problems will students be more successful with continuous quantities or with discrete ones (Q2)? Third, will students use more additive strategies in discrete proportional problems or continuous ones (Q3)? Fourth, will students use more proportional strategies in continuous additive problems or discrete ones (Q4)? And fifth, will the effects of the nature of quantities disappear with students' age (Q5)?

RESULTS

Description of the General Trends

Table 3 shows the percentages of correct answers to the proportional and additive problems in the different age groups. In contrast to HYPO 1, students were generally more successful in additive problems (44.3%) than proportional ones (40.3%). Although the difference was not very large, a repeated measures logistic regression analysis showed that it was significant, $\chi^2(1, N = 558) = 6.248$, $p = 0.012$.

However, there was also a significant "type of problem" × "grade" interaction effect, $\chi^2(3, N = 558) = 99.074$, $p < 0.001$, revealing a very strong increase in performance in the proportional problems from 1st to 4th grade (from 13.8% to 72.3% to) and a mod-

Table 3. Percentages of correct answers in the proportional (P) and additive problems (A) with integer (I) and non-integer (N) structure and with discrete (D) and continuous (C) nature of quantities for each grade

Number structure	1st	2nd	3rd	4th	Total
P-I	22.0	33.0	55.5	75.0	46.4
P-N	5.5	20.0	42.0	69.5	34.3
Total	13.8	26.5	48.8	72.3	40.3
A-I	46.0	48.5	37.5	27.0	39.8
A-N	56.5	61.0	44.5	33.0	48.8
Total	51.3	54.8	41.0	30.0	44.3
Nature of quantities	1st	2nd	3rd	4th	Total
P-D	15.0	26.0	49.0	71.5	40.4
P-C	12.5	27.0	48.5	73.0	40.3
Total	13.8	26.5	48.8	72.3	40.3
A-D	50.0	54.5	41.0	30.5	44.0
A-C	52.5	55.0	41.0	29.5	44.5
Total	51.3	54.8	41.0	30.0	44.3

erate decrease in the additive problems (from 51.3% to 30.0%). The 1st and 2nd grade secondary school students were much more successful in additive problems than proportional problems, whereas 3rd and 4th grade students were more successful in proportional problems than additive ones.

There was a decrease in the use of additive strategy in proportional problems (Table 4) in secondary school (from 52.0% in 1st grade to 21.2% in 4th grade), resulting in a significant main effect for "grade", $\chi^2(3, N = 558) = 8.995$, $p = 0.029$. While the difference in the use of additive strategy in proportional problems was not significant between 1st and 2nd grade (52.0% vs. 47.0%), differences between 2nd and 3rd grade (47.0% vs. 34.2%) and between 3rd and 4th grade (34.2% vs. 21.2%) were significant. Furthermore, there was a strong increase with age in the use of proportional strategy in additive problems (from 11.0% in 1st grade to 63.7% in 4th grade). This main effect of "grade" was significant, $\chi^2(3, N = 558) = 26.567$, $p < 0.001$. The difference in the use of proportional strategy in additive problems was always significant between consecutive grades: between 1st and 2nd grade (11.0% vs. 20.0%), between 2nd and 3rd grade (20.0% vs. 43.5%), and between 3rd and 4th grade (43.5% vs. 63.7%). These results suggest that, as in Van Dooren et al.'s (2005) study with Flemish primary school students, Spanish secondary school students increasingly apply proportional solution methods to non-proportional (additive) problems.

Effect of "Number Structure"

We found a "type of problem" × "number structure" interaction effect in performance, $\chi^2(1, N = 558) = 104.604$, $p < 0.001$. As shown in Table 3, in proportional problems students were more successful with the integer versions (46.4%) than with the non-integer versions (34.3%), whereas in additive problems, the non-integer versions (48.8%) were solved better than the integer versions (39.8%). Thus HYPO 2A and HYPO 2C were both confirmed.

With regard to the strategy used, Table 4 shows that globally students used the additive strategy more frequently in the non-integer versions (41.6%) than the integer versions (35.6%) of the proportional problems. Statistical analysis revealed that this main effect of "number structure" on the use of the additive strategy, was significant, $\chi^2(1, N = 558) = 51.974$, $p < 0.001$. Thus HYPO 2B was confirmed as well.

Table 4. Percentages of students' use of the additive strategy in the proportional (P) problem and the proportional strategy in additive (A) problem with integer (I) and non-integer (N) structure for each grade

Additive Strategy	1st	2nd	3rd	4th	Total
P-I	48.0	42.5	31.5	20.5	35.6
P-N	56.0	51.5	37.0	22.0	41.6
P (total)	52.0	47.0	34.2	21.2	38.6
Proportional Strategy					
A-I	18.5	26.0	49.0	67.5	40.3
A-N	3.5	14.0	38.0	60.0	28.9
A (total)	11.0	20.0	43.5	63.7	34.6

In relation to additive problems, students used more proportional strategies in the integer (40.3%) than the non-integer versions (28.9%). This “number type” effect $\chi^2(1, N = 558) = 90.519, p < 0.001$ also turned out to be significant (HYPO 2D).

Finally, our results also confirmed HYPO 2E. The differences in students’ performance between integer and non-integer versions became smaller with age (see Table 3). The analysis revealed a “grade” × “type of problem” × “number type” interaction effect on students’ performance, $\chi^2(3, N = 558) = 19.352, p < 0.001$. For the proportional problems, the difference in performance between the integer and non-integer versions was very strong and significant in 1st grade (22.0% vs. 5.5% correct answers) and 2nd grade (33.0% vs. 20.0%), whereas it was smaller but still significant in 3rd (55.5% vs. 42.0%) and 4th grade (75.0% vs. 69.5%). Also, for the additive problems, the difference in performance between the integer and non-integer versions was strong and significant in 1st grade (46.0% vs. 56.5%) and 2nd grade (48.5% vs. 61.0%), less strong but still significant in 3rd (37.5% vs. 44.5%) and 4th grades (27.0% vs. 33.0%).

Effect of “Nature of Quantity”

With regard to discrete versus continuous nature of quantities, there was no significant main effect on students’ performance ($\chi^2(1, N = 558) = 0.542, p = 0.462$) nor an interaction effect between the nature of quantity factor and grade, type of problem or number structure ($\chi^2(3, N = 558) = 0.210, p = 0.976$; $\chi_c(1, N = 558) = 0.135, p = 0.713$ and $\chi^2(1, N = 558) = 1.304, p = 0.254$, respectively). Table 3 shows the percentages of correct answers in the discrete and continuous proportional and additive problems, separated out by grade. The data, indeed, indicate no differences in students’ success in the proportional problems (40.4% in discrete vs. 40.3% in continuous), nor in the additive problems (44.0% in discrete vs. 44.5% in continuous). So, students were not more successful in discrete proportional problems than continuous proportional ones and not more successful in discrete additive problems than continuous additive ones (questions Q1 and Q2).

The data on the use of additive strategies in the discrete or continuous proportional

Table 5. Percentages of students’ use of the additive strategy in the proportional (P) problem and the proportional strategy in the additive (A) problem with discrete (D) and continuous (C) quantities

Additive Strategy	1st	2nd	3rd	4th	Total
P-D	52.5	47.5	35.0	22.0	39.3
P-C	51.5	46.5	33.5	20.5	38.0
P (total)	52.0	47.0	34.2	21.2	38.6
Proportional Strategy					
A-D	12.5	20.5	44.5	63.0	35.1
A-C	9.5	19.5	42.5	64.5	34.0
A (total)	11.0	20.0	43.5	63.7	34.6

problems and on the use of proportional strategies in the additive discrete or continuous problems (Table 5) show that the factor "nature of quantity" had no impact (χ^2(1, N = 558) = 0.155, p = 0.694 and χ^2(1, N = 558) = 1.500, p = 0.221, respectively). With regard to the use of the additive strategy, 39.3% of students used it in the discrete proportional problem and 38.0% of students in the continuous proportional one. With respect to the use of proportional strategies in additive problems, 35.1% of students used them in the discrete additive problem and 34.0% in the continuous additive one. So, students did not use the additive strategy more frequently in discrete proportional problems than in continuous ones and did not use proportional strategies more often in discrete additive problems than in continuous ones (questions Q3 and Q4).

The research question Q5 is related to the effect of discrete and continuous variables in relation to age. The absence of this effect (χ^2(3, N = 558) = 2.475, p = 0.480) was not surprising, given that there was no effect of this variable overall.

CONCLUSIONS AND DISCUSSION

The study links two important aspects of the proportional reasoning literature. On the one hand, research indicates that some variables influence students' performance and strategies in proportional problems, and more particularly elicit students' additive strategies and errors (Harel, Behr, 1989; Jeong et al., 2007; Tourniaire, Pulos, 1985; Van Dooren et al., 2009). On the other hand, there are studies that have shown students' over-use of proportional strategies in problems that actually have a non-proportional (e.g., an additive) structure (De Bock et al., 2007; Van Dooren et al., 2005). In the present investigation we extend Van Dooren et al.'s (2009) study on how the variable "number structure" (non-integers vs. integer ratios) affects Flemish primary school students' performance, to consider Spanish secondary school students. We investigate how the trends identified by Van Dooren et al. (2009) continue in secondary school students. Additionally, we also study a new aspect: the role of the "nature of quantities" (discrete or continuous). We do this because the literature on the role of this variable was inconsistent, and even nonexistent with respect to the over-use of proportional methods in non-proportional situations.

Globally, our results point out that 1st and 2nd grade secondary school students were more successful in solving additive problems than proportional problems, whereas 3rd and 4th grade secondary school students were more successful in proportional problems than additive ones. Possibly, this manifests an influence of the curriculum. For example, when students learn the Rule of Three algorithm in the 1st grade of secondary school as an algorithm that can be applied in many problems "with similar structure", they used it in all the problems, including the additive ones, without taking into account whether there is a multiplicative or an additive relationship between the quantities. From this point of view, students increase their success in solving proportional problems, but decrease it in additive ones. This point underlines the importance of instruction to promote the development of proportional reasoning that goes along with paying attention to where this kind of reasoning is applicable.

With regard to the impact of the integer or non-integer number structure, students were, as expected, more successful in proportional problems with integer ratios than non-integer ratios. This trend indicates that non-integer ratios lead students to apply more additive strategies in proportional problems. For additive problems, performance was better in problems with non-integer ratios than for additive problems with integer ratios. This trend was due to students' tendency to apply more proportional methods to additive problems with integer ratio. These effects were particularly strong in 1st and 2nd grade secondary school students. They became less influential in 3rd and 4th grade secondary school students but certainly did not disappear. A possible explanation for the influence of the number structure is that when ratios are integer, students can rely on the identification of whole number relations between numbers (i.e., knowledge that one number is the double, triple... of another), and then take the double (triple) of the third number in the problem too. With non-integer ratios, however, they have to *calculate* the ratio and apply the result of this calculation to the third number (Freudenthal, 1983). Our results concerning the effect of number structure obtained with Spanish secondary school students expanded those reported by Van Dooren et al. (2009) with Flemish primary school students.

With regard to the discrete or continuous nature of the quantities, we did not find any difference in students' performance in the strategies used by students nor in the interaction with integer or non-integer number structure. As mentioned above, literature about the effect of this variable was inconclusive. It may be that we did not find any effect of this task variable because we used missing-value word problems instead of comparison problems, for which the effect has been shown in previous research (Boyer et al., 2008; Jeong et al., 2007). Another reason may be that our word problems (even the non-integer versions) always had integer outcomes. Maybe we would have observed a significant difference if we had included word problems with non-integer ratios and non-integer outcomes as well, such as "*Peter and Tom are loading boxes in a truck. They started together but Tom loads faster. When Peter has loaded 3 boxes, Tom has loaded 5 boxes. If Peter has loaded 7 boxes, how many boxes has Tom loaded?*" In those problems, proportional solutions to discrete problems make little or no sense (e.g., one cannot load $(5 / 3) \times 7 = 11.6$ boxes in a truck), whereas they are perfectly reasonable for the versions with continuous quantities, as in "*Ann and Rachel are skating. They started together but Rachel skates faster. When Ann has skated 10 m, Rachel has skated 15 m. If Ann has skated 21 m, how many meters has Rachel skated?*" (One can skate $(15 / 10) \times 21 = 31.5$ meters). A third element that may explain this result is that we involved secondary school students, whereas in previous research participants were mostly primary school students (Boyer et al., 2008; Jeong et al., 2007; Spinillo, Bryant, 1999).

For further research, we could look at other task variables and their interaction with the nature of quantities (discrete or continuous), such as the use of comparison problems or problems with non-integer outcomes. Furthermore, we could study if task switching causes the over-use of the proportional or additive approach. For instance, comparing the performance in an additive problem depending on whether it is embedded in a test full of proportional problems, or in a test full of additive, or buffer problems.

Finally, our findings have implications for teaching practice. We have observed that secondary school students take into account the numbers in a missing-value problem to decide which strategy they will use to solve it. But it is unclear if students are consciously aware of the role of the number structure in their choice of one strategy or another. The given numbers can be considered as stimuli that prime the strategy for solving the task. In fact, integer ratios might prime the proportional strategy merely at the level of number perception, while students read the task. It is a question for further research whether this is indeed the case.

Our study suggests, as in Van Dooren et al. (2009, p. 207), that "the classroom teaching of proportionality might benefit from explicitly discussing the criteria that students use when deciding on the appropriateness of proportional solution methods". One necessary instructional focus would be on the relations between quantities, recognizing that the transition from the double, triple… to fractions could be difficult for the students while helping to discriminate additive structure from multiplicative structure. So, teaching should support the construction of conceptual understanding of ratio and should encourage overcoming a simplistic mechanical treatment of proportion as the manipulation of symbols (Lamon, 1999). To do that, it seems advisable to propose other proportional problems in other formats, for example, comparing two given ratios in quantitative and qualitative ways.

Although this study has found that the variable nature of quantities does not have any influence on secondary school students' performance, this variable could be taken into account in classroom practice. For instance, discrete problems with a non-integer solution (or with non-integer middle operations) can be very interesting to discuss with pupils because their solution and/or some of the problem-solving steps have no sensible meaning in real life, as we have mentioned before. By discussing these problems, students might develop a more abstract understanding of the multiplicative relations they are dealing with.

Furthermore, students' over-use of proportionality is often caused by their practice to solve missing-value proportional problems in the classroom (the majority of time using the algorithm of the Rule of Three). This practice provides them speed and fluency to solve these proportional problems but it is possible that it does not provide them the necessary understanding of the problem structure. It seems necessary to solve and discuss in the classroom other types of proportional problems, such as numerical or qualitative comparison problems, and contrast them with similar non-proportional problems (such as additive problems).

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VPLYV ŠTRUKTÚRY ČÍSIEL A CHARAKTERU VELIČÍN NA PROPORCIONÁLNE UVAŽOVANIE U 12-16-ROČNÝCH ŠTUDENTOV

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Súhrn: Štúdia skúma vplyv štruktúry čísiel (celočíselné a neceločíselné pomery) a charakteru veličín (diskrétne vs. kontinuálne) na výkon študentov a na stratégie, ktoré používajú pri riešení proporcionálnych a doplnkových slovných úloh. 551 12-16-ročných študentov vyplňalo test, ktorý pozostával z proporcionálnych a doplnkových slovných úloh, v ktorých sa manipulovalo s vyššie spomínanými premennými. Výsledky ukázali, že štruktúra čísiel ovplyvňovala výkon študentov, ktorí používali rôzne stratégie pri riešení proporcionálnych a doplnkových slovných úloh v jednotlivých ročníkoch. Zistilo sa však, že charakter veličín nemá na riešenie slovných úloh žiaden vplyv.